# 數學系課程核心教材內容

課程名稱:(中	文) 近代幾何學	開課單位	數學碩博班		
(英文) Introduction to Modern Geometry(I)				課程代碼	2405007
學分數	3	必/選修	必修	開課年級	_

教學目標: It will be of fundamental importance for students intending to specialize in differential geometry, geometric analysis, global PDEs, differential topology or dynamical systems. This will be a fairly standard course in smooth manifolds and their geometry.

課程概述: The course is designed for a variety of graduate students having widely varying kinds of preparation and interests. This is the part I of a two-semester course, covering the foundations of Differential and Riemannian Geometry.

先修科目或先備能力:The only prerequisite is an upper-division analysis course. Courses on the geometry of curves and surfaces, or on manifolds and topology, will be useful but not necessary.

## 建議參考書目

- 1. M. do Carmo, Riemannian Geometry, Birkhauser.
- 2. F. Warner, Foundations of Differentiable manifolds and Lie group.
- 3. Peter Petersen, Riemannian geometry, Springer.

### 課程大綱

單元主題	內容綱要	上課週數
1 Manifolds	Differentiable manifolds, Vector fileds, Tensor fields, Differential forms	4 weeks
2 Riemannian metrics	Riemannian manifolds, Local representations of metrics, Doubly warped product spaces, More examples	3 weeks
3 Connections	Affine connections, Levi-Civita connections, moving frames	3 weeks
4 Geodesic	Geodesics and distance, The exponential map, Convex neighborhoods	3 weeks
5 Curvature	Curvature, Sectional curvature, Ricci curvature, Scalar Curvature, Tensor calculus	3 weeks

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課程名稱:(中	文) 近代幾何學	開課單位	數學碩博班		
(英文) Introduction to Modern Geometry(II)				課程代碼	2405008
學分數	3	必/選修	必修	開課年級	_

教學目標: It will be of fundamental importance for students intending to specialize in differential geometry, geometric analysis, global PDEs, differential topology or dynamical systems. This will be a fairly standard course in smooth manifolds and their geometry.

課程概述: The course is designed for a variety of graduate students having widely varying kinds of preparation and interests. This is the part II of a two-semester course, covering the foundations of Differential and Riemannian Geometry.

先修科目或先備能力:The only prerequisite is an upper-division analysis course. Courses on the geometry of curves and surfaces, or on manifolds and topology, will be useful but not necessary.

## 建議參考書目

- 1. M. do Carmo, Riemannian Geometry, Birkhauser.
- 2. Cheeger and Ebin, Comparison theorems in Riemannian geometry.
- 3. Peter Petersen, Riemannian geometry, Springer.

#### 課程大綱

單元主題	內容綱要	上課週數
1 Jacobi fields	Jacobi equation, Conjugate points	3 weeks
2 Isometric immersions	Hypersurfaces, the second fundamental form, structure equations	3 weeks
3 Complete manifolds	Completeness, the Hopf-Rinow theorem, the Hadamard theorem	2 weeks
4 Spaces of constant curvature	Space forms	3 weeks
5 Variations of energy	The first and second variations of energy, Bonne-Myers theorem, Synge-Weinstein theorem	2 weeks
6 The Rauch comparison theorem	The theorem of Rauch, Focal points and an extension of Rauch's theorem	3 weeks