## 數學系課程核心教材內容

| 課程名稱:(中 | 文)複變函數論       | 開課單位 | 學士班     |      |   |
|---------|---------------|------|---------|------|---|
| (英      | 文) Complex An | 課程代碼 | 2103101 |      |   |
| 學分數     | 3             | 必/選修 | 必選      | 開課年級 | 3 |

教學目標: Understanding and performing calculation of complex differentiation and integration and their applications.

課程概述:Introduction to algebraic and geometric properties of complex numbers and calculus of complex functions. Analyticity of complex functions and complex integration theory.

先修科目或先備能力:Calculus, Advanced calculus

建議參考書目

- 1. Mathews and Howell, Complex Analysis
- 2. Pennisi and Gordon, Elements of Complex Variables

## 課程大綱

| 單元主題                            | 內容綱要   | 上課週數  |
|---------------------------------|--|-------|
| Complex numbers                 | The algebra and geometry of complex numbers. The topology of complex numbers. Bolzano-Weierstrass theorem. Jordan curve theorem.   | 2     |
| Complex functions               | Functions and linear mappings. Limit and continuity. The power, root and reciprocal transformations. Branches of functions.  | 2     |
| Analytic and harmonic functions | Differentiability and analyticity of complex functions. The Cauchy-Riemann equations. Harmonic functions and their conjugates.   | 2 ~ 3 |
| Complex sequences and series    | Limit of complex sequences and series. Geometry series and convergence theorems. Power series functions.   | 1 ~ 2 |
| Elementary functions            | The complex exponential function. The complex logarithmic function. Complex exponents. Trigonometric and hyperbolic functions. Inverse trigonometric and inverse hyperbolic functions (optional).                  | 2     |
| Complex integration             | Contour and contour integrals. The Cauchy-Goursat theorem. The fundamental theorem of integration. Integration representations for analytic functions. Morera's theorem, Liouville's theorem, Cauchy's inequality. | 4     |
| Taylor and Laurent series       | Uniform convergence. Taylor series representation. Laurent series representation. Singularities, zeros and poles. Applications of Taylor and Laurent series.   | 2~3   |
| Elementary residue calculus     | The residue theorem. Trigonometric integrals. Improper integrals of rational functions.  | 1 ~ 2 |

## 數學系課程核心教材內容

| 課程名稱:(中                    | 文)複變函數(二 | 開課單位 | 學士班 |      |         |
|----------------------------|----------|------|-----|------|---------|
| (英文) Complex analysis (II) |          |      |     | 課程代碼 | 2103102 |
| 學分數                        | 3        | 必/選修 | 選修  | 開課年級 | 3       |

教學目標: Good skill in performing integration by residue. Application of residues. Understanding and performing calculations of conformal mapping, and Fourier series and Laplace transforms.

課程概述: The course continues the development of residue theory to the intermediate level and its more advanced applications. The course also discusses conformal mappings, and Fourier and Laplace transforms and their applications.

先修科目或先備能力:Calculus, Advanced calculus, Complex analysis I

建議參考書目

- 1. Antimirov, Kolyshkin, and Vaillancourt, Complex Variables
- 2. Pennisi and Gordon, Elements of Complex Variables

## 課程大綱

| 單元主題                                    | 內容綱要  | 上課週數 |
|---|---|------|
| Calculus of residue                     | Singular points of analytic functions. The residue theorem. Trigonometric integrals. Definite integrals of rational function over the whole real line. Definite integrals of rational functions time sine or cosine. Definite integrals of rational functions time exponential functions. Definite integrals of rational functions time a power function. | 3    |
| Intermediate residue calculus           | Improper definite integrals of rational functions over the positive real line. Poisson integrals, Fresnel integrals.  | 3    |
| Application of residue theory 1         | Counting zeros and poles of meromorphic functions. The argument principle. Rouche's theorem.  | 2    |
| Application of residue theory 2         | Simple pole expansion of meromorphic functions. Mittag-<br>Leffler theorem. Infinite product expansion of entire<br>functions.  | 2    |
| Conformal mapping                       | The linear transformation. The linear fractional transformations. The invariance of cross ratio. Symmetry with respect to a circle. Jukowski airfoil. Schwartz-Christoffel transformation.  | 3    |
| Fourier series and<br>Laplace transform | Fourier series. The Dirichlet problem for the unit disk. The Fourier transform. The Laplace transform. Inverting the Laplace transform. Convolution.  | 3    |